



Province of the
EASTERN CAPE
EDUCATION

MATHEMATICS

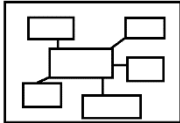



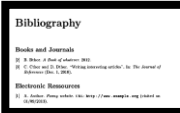

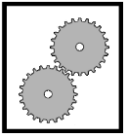

2025 SPRING SCHOOL

GRADE 12

GUIDE FOR TEACHERS AND LEARNERS



ICON DESCRIPTION

 <p>MIND MAP</p>	 <p>EXAMINATION GUIDELINE</p>	 <p>CONTENTS</p>	 <p>ACTIVITIES</p>
 <p>BIBLIOGRAPHY</p>	 <p>TERMINOLOGY</p>	 <p>WORKED EXAMPLES</p>	 <p>STEPS</p>



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FINANCE, GROWTH AND DECAY

Simple Growth

Formula: $A = P(1 + n \cdot i)$

A = The final amount

P = The initial amount

n = number of years

i = interest rate in decimals

Compound Growth

Formula: $A = P(1 + i)^n$

A = The final amount

P = The initial amount

n = number of years

i = interest rate in decimals

Worked Example (November 2010 Q7.1)

At what annual percentage interest rate, compounded quarterly, should a lump sum be invested in order for it to double in 6 years?

Solution

$$A = P(1 + i)^n$$

$$2P = P \left(1 + \frac{r}{400} \right)^{6 \times 4}$$

$$2 = \left(1 + \frac{r}{400} \right)^{24}$$

$$1 + \frac{r}{400} = 2^{\frac{1}{24}}$$

$$r = 400 \left(2^{\frac{1}{24}} - 1 \right)$$

$$r = 400 \left(2^{\frac{1}{24}} \right) - 400$$

$$r = 11,72\% \text{ p.a.}$$

Simple Decay**(Straight line depreciation)****Formula: $A = P(1 - n \cdot i)$** A = The final amount P = The initial amount n = number of years i = interest rate in decimals***N.B the initial amount is bigger than the final amount*****Compound Decay****(reducing balance depreciation)****Formula: $A = P(1 - i)^n$** A = The final amount P = The initial amount n = number of years i = interest rate in decimals***N.B the initial amount is bigger than the final amount*****Solving for n**

$$A = P(1 \pm i)^n$$

$$(1 \pm i)^n = \frac{A}{P}$$

$$\log(1 \pm i)^n = \log \frac{A}{P}$$

$$n \log(1 \pm i) = \log A - \log P$$

$$\therefore n = \frac{\log A - \log P}{\log(1 \pm i)}$$

Worked Example (November 2011 Q7.1)

How many years will it take for an article to depreciate to half its value according to the reducing-balance method at 7% per annum?

Solution

$$A = P(1 - i)^n$$

$$\frac{P}{2} = P(1 - 0,07)^n$$

$$\frac{1}{2} = 0,93^n$$

$$\log_{0,93} \frac{1}{2} = n$$

$$n = 9,55 \text{ years}$$

Nominal and Effective Interest rates

Nominal Rate

The rate quoted, and compounding periods are different:

e.g. 10% p.a. compounded quarterly

Effective Rate

The rate quoted, and compounding periods are the same:

e.g. 10% p.a. compounded annually

5% per month compounded monthly

Formula to convert from nominal rate to effective annual rate (and vice versa)

$$1 + i_{eff} = \left(1 + \frac{i_{nom}}{m} \right)^m$$

i_{eff} = **effective annual rate**

i_{nom} = **nominal rate**

m = **number of compounding periods per year**

Future Value Annuity

Formula: $F = \frac{x[(1+i)^n-1]}{i}$

F = Future value

x = fixed regular payments

n = number of payments

i = interest rate in decimals

When there is “ x ” immediate payment made, and the last payment is made at the end of the period:

Use the following formula: $F = \frac{x[(1+i)^{n+1}-1]}{i}$

When there is an immediate payment made of an amount that is not x , say t , and the last payment is made at the end of the period:

Use the following formula: $F = t(1+i)^n + \frac{x[(1+i)^n-1]}{i}$

When payments are made at the beginning of each period or when payments are made at the end of each period and the last payment is made, for an example 1 month before the end of the period if interest is compounded monthly:

Use the following formula: $F = \frac{x[(1+i)^n-1]}{i} \times (1+i)^n$

Sinking Fund

Sinking fund is an amount that is invested to replace something (e.g. Vehicle, Machinery) in future. We use future value annuity to save money in regular intervals for the money to be used in future.

N.B Sinking fund (when the amount is not stated)

$= \text{New price after inflation} - \text{Book value/Scrap value}$

Worked Example (November 2012 Q7.1)

7.1 A business buys a machine that costs R120 000. The value of the machine depreciates at 9% per annum according to the diminishing-balance method.

7.1.1 Determine the scrap value of the machine at the end of 5 years.

7.1.2 After five years the machine needs to be replaced. During this time, inflation remained constant at 7% per annum. Determine the cost of the new machine at the end of 5 years.

7.1.3 The business estimates that it will need R90 000 by the end of five years. A sinking fund for R90 000, into which equal monthly instalments must be paid, is set up. Interest on this fund is 8,5% per annum, compounded monthly. The first payment will be made immediately and the last payment will be made at the end of the 5-year period.

Calculate the value of the monthly payment into the sinking fund.

Solution

7.1.1	$A = P(1 - i)^n$ $= 120\,000(1 - 0,09)^5$ $= R74\,883,86$
7.1.2	$A = P(1 + i)^n$ $= 120\,000(1 + 0,07)^5$ $= R168\,306,21$
7.1.3	<p>Sinking fund needed: $F_v = R\,90\,000$</p> $F_v = \frac{x[(1 + i)^n - 1]}{i}$ $90\,000 = \frac{x \left[\left(1 + \frac{0,085}{12} \right)^{61} - 1 \right]}{\frac{0,085}{12}}$ $x = R\,1\,184,68$

Present Value Annuity

Formula: $P = \frac{x[1-(1+i)^{-n}]}{i}$

P = Present value (loan amount)

x = fixed regular payments

n = number of payments

i = interest rate in decimals

Interest paid

Interest amount paid = All payments made – loan amount

Balance on the loan

$$\text{Balance} = P(1+i)^n - \frac{x[(1+i)^n - 1]}{i}$$

OR

$$\text{Balance} = \frac{x[1-(1+i)^{-n}]}{i}$$

$n \rightarrow$ number of payments left

Worked Example (November 2010 Q7.2)

7.2 Timothy buys furniture to the value of R10 000. He borrows the money on 1 February 2010 from a financial institution that charges interest at a rate of 9,5% p.a. compounded monthly. Timothy agrees to pay monthly instalments of R450. The agreement of the loan allows Timothy to start paying these equal monthly instalments from 1 August 2010.

7.2.1 Calculate the total amount owing to the financial institution on 1 July 2010.

7.2.2 How many months will it take Timothy to pay back the loan?

7.2.3 What is the balance of the loan immediately after Timothy has made the 25th payment?

Solution

7.2.1	$A = 10000 \left(1 + \frac{0,095}{12} \right)^5$ $= R\ 10\ 402,15$
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7.2.2

$$10402,15 = \frac{450 \left[1 - \left(1 + \frac{0,095}{12} \right)^{-n} \right]}{\frac{0,095}{12}}$$

$$0,183000787 = 1 - \left(1 + \frac{0,095}{12} \right)^{-n}$$

$$\left(1 + \frac{0,095}{12} \right)^{-n} = 0,816999213$$

$$\log \left(1 + \frac{0,095}{12} \right)^{-n} = \log 0,816999213$$

$$-n \log \left(1 + \frac{0,095}{12} \right) = \log 0,816999213 \dots$$

$$n = 25,63151282 \dots$$

$$n = 25,63 \text{ months}$$

$$n = 26$$

7.2.3	<p>Balance outstanding after 25 months</p> $= 10402,15 \left(1 + \frac{0,095}{12} \right)^{25} - \frac{450 \left[\left(1 + \frac{0,095}{12} \right)^{25} - 1 \right]}{\frac{0,095}{12}}$ $= R\ 282,36$
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OR

$$n = 25,6315128204 \dots - 25$$

$$= 0,6315128204 \dots$$

Balance Outstanding after 25 months

$$= \frac{450 \left[1 - \left(1 + \frac{0,095}{12} \right)^{-0,631512804} \right]}{\frac{0,095}{12}}$$

$$= R\ 282,36$$

ACTIVITIES

Finance, Growth and Decay

(May/June 2024)

QUESTION 7

- 7.1 Six years ago, Thabo bought a phone for R13 000. The value of the phone depreciated annually according to the reducing-balance method. The value of the phone is now R8 337,75. Calculate the annual rate of depreciation. (3)
- 7.2 Eric and Thandi need to save R80 000 each to go on a holiday at the end of December 2027.
- Thandi decides that she will start saving at the end of January 2025. She will make 36 monthly deposits into a savings account that pays interest at 8,6% p.a., compounded monthly. The deposit will be made at the end of each month.
 - Eric calculates that if he makes 48 deposits of R1 402,31, starting at the end of January 2024, he will have enough money to go on holiday. He will make his deposits into a savings account at the end of each month. The savings account pays interest at 8,6% p.a., compounded monthly.
- Calculate the difference between the total amount that Eric and Thandi will deposit into their respective savings accounts over the given period. (4)
- 7.3 Lesibana was granted a loan of R225 000. The rate of interest for the loan is 9% p.a., compounded monthly. Lesibana will make monthly payments of R5 500, starting exactly four months after the loan was granted. How many payments will Lesibana make to settle the loan? (6)
- [13]

(May/June 2023)

QUESTION 6

- 6.1 A company bought a photocopier for R150 000 on 1 July 2022. They will use the old photocopier as a trade-in when they replace it with a similar new photocopier in 5 years' time on 30 June 2027.
- 6.1.1 The average rate of inflation over the next 5 years will be 6,5% p.a. Determine the price of a similar new photocopier in 5 years' time. (2)
- 6.1.2 Calculate the trade-in value of the old photocopier after 5 years, if it depreciates at a rate of 9% p.a. on a straight-line method. (2)
- 6.1.3 The company set up a sinking fund to cover the replacement cost of the new photocopier. The fund earns interest at the rate of 7,85% p.a., compounded monthly. The company made its first monthly deposit on 31 July 2022 and will continue to do so until 31 May 2027, one month prior to the new photocopier being bought. How much should be deposited at the end of each month so that the company will be able to buy the new photocopier? (4)
- 6.2 Today, Andrew borrowed R200 000 from a bank. The bank charges interest at 5,25% p.a., compounded quarterly. Andrew will make repayments of R6 000 at the end of every 3 months. His first repayment will be made in 3 months from now. How long, in years, will it take Andrew to settle the loan? (5)
[13]

(May/June 2022)

QUESTION 7

- 7.1 How many years will it take for an investment to double in value, if it earns interest at a rate of 8,5% p.a., compounded quarterly? (4)
- 7.2 A company purchased machinery for R500 000. After 5 years, the machinery was sold for R180 000 and new machinery was bought.
- 7.2.1 Calculate the rate of depreciation of the old machinery over the 5 years, using the reducing-balance method. (4)
- 7.2.2 The rate of inflation for the cost of the new machinery is 6,3% p.a. over the 5 years. What will the new machinery cost at the end of 5 years? (2)
- 7.2.3 The company set up a sinking fund and made the first payment into this fund on the day the old machinery was bought. The last payment was made three months before the new machinery was purchased at the end of the 5 years. The interest earned on the sinking fund was 10,25% p.a., compounded monthly. The money from the sinking fund and the R180 000 from the sale of the old machinery was used to pay for the new machinery.
- Calculate the monthly payment into the sinking fund. (5)
[15]

STATISTICS AND REGRESSION

Measures of Central Tendency for Ungrouped Data

$\text{Mean} = \frac{\text{sum of all values}}{\text{total number of values}}$ $\bar{x} = \frac{\sum x}{n}$	Where : \bar{x} = mean $\sum x$ = sum of all values n = number of values
---	---

Mode

The mode is the value that appears most frequently in a set of data points.

Median

The median is the middle number in a set of data points. position of median = $\frac{1}{2}(n+1)$

N.B Data must be arranged in ascending order before calculating the median

Range as a Measure of Dispersion

Range = Biggest Values – Smallest Value

Measures of Central Tendency for Grouped Data

$\text{Estimated Mean} = \frac{\sum f \cdot x_i}{\sum f}$	$f \rightarrow$ frequency $x_i \rightarrow$ the class midpoint Calculating x_i $x_i = \frac{1}{2} (\text{lower class limit} + \text{upper class limit})$
Modal Class	The class with the highest frequency

Class With Median

- Position of median for grouped data $\rightarrow \frac{1}{2}n$
- Calculating class with median \rightarrow add the frequencies from the top to locate the class with the median:

Time(hours)	Frequency	
$0 \leq x < 1$	5	↓ 5 values
$1 \leq x < 2$	9	↓ 14 values
$2 \leq x < 3$	12	↓ 26 values
$3 \leq x < 4$	6	
$n = \Sigma f = 32$		

The median lies here

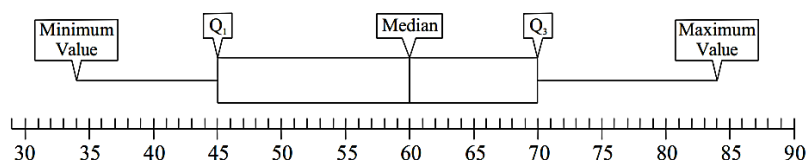
N.B position of median $= \frac{1}{2}n = \frac{1}{2}(32) = 16$

Five Number Summary and Box and Whisker Plot**FIVE NUMBER SUMMARY**

1. Minimum value
2. Lower quartile Q_1
3. Median Q_2
4. Upper quartile Q_3
5. Maximum value

BOX AND WHISKER PLOT

A box and whisker plot is a visual representation of the five number summary.

**Identifying Outliers**

- Any data item that is

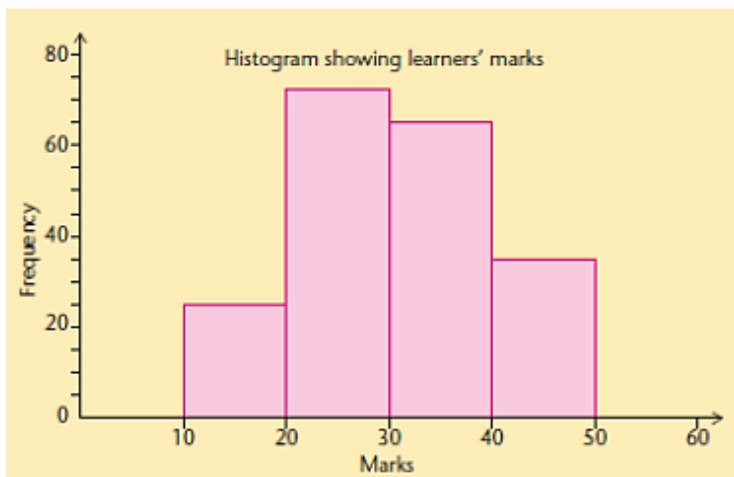
Less than $Q_1 - 1,5 \times IQR$
OR
More than $Q_3 + 1,5 \times IQR$
is an outlier.

Histograms

A histogram gives us a visual interpretation of **GROUPED DATA**. It looks very similar to a **bar graph**, but there are **NO** gaps between the bars.

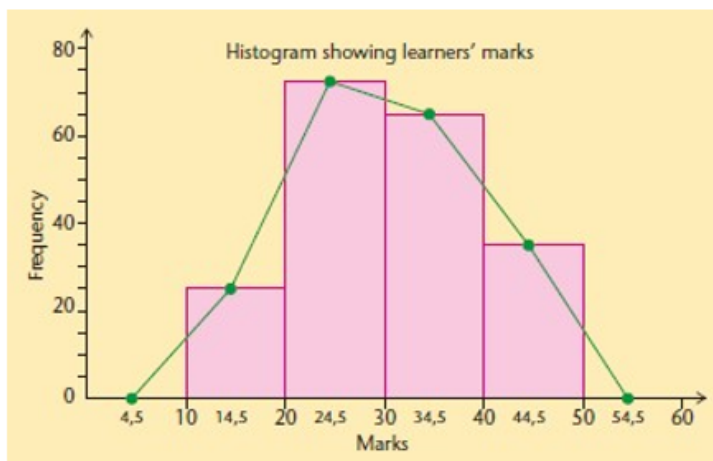
In HISTOGRAM INCLUDE THE FOLLOWING

- Title on top that describe what is contained in histogram
- Group/class intervals in x axis
- Frequency in y axis
- Bars with no gaps in between

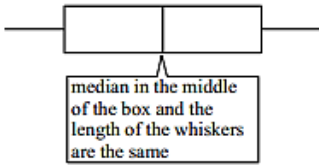
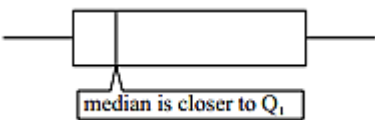
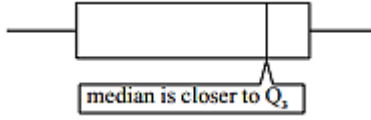
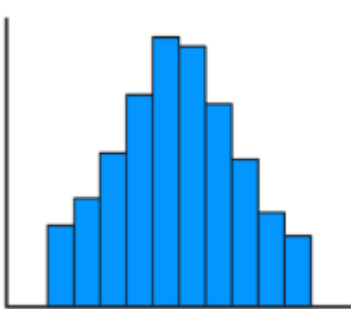
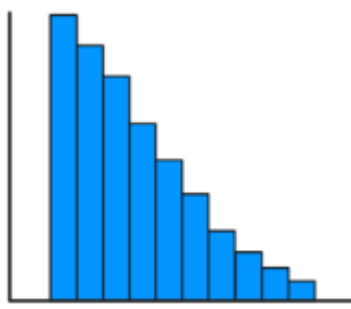
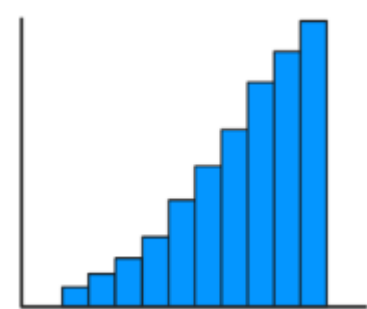
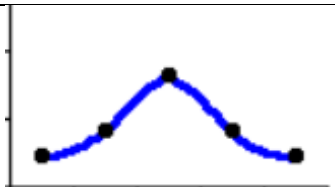
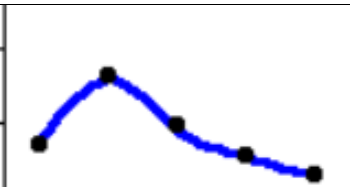
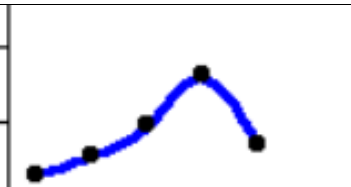


Frequency Polygons

Drawn from **HISTOGRAM** by joining the **midpoints** of the top of the columns of the histogram. At the ends, extend line to the midpoints of class below lower values and the midpoint of the class above upper value to touch x axis(**grounded**)



Distribution of The Data

Symmetrical	Positively Skewed (Skewed to The Right)	Negatively Skewed (Skewed to The Left)
 <p>median in the middle of the box and the length of the whiskers are the same</p>	 <p>median is closer to Q_1</p>	 <p>median is closer to Q_3</p>
		
		
Mean = Median	Mean > Median	Mean < Median

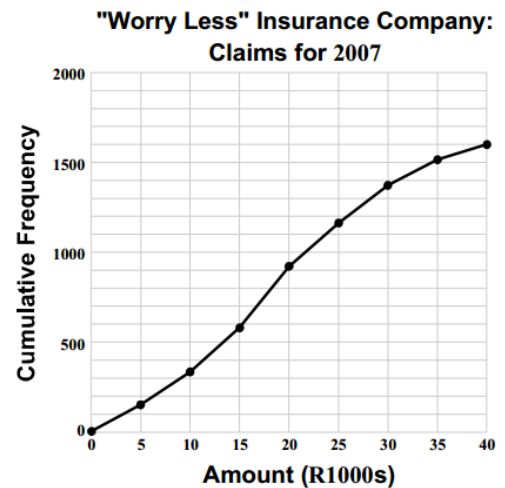
Ogives (Cumulative Frequency Curves)

To find the **cumulative frequency**,

- Add up the frequencies as you go down the frequency table.
- The last value of **cumulative frequency** must be equal to the total sum of all frequencies.

Always remember when drawing cumulative frequency curve, points to be plotted are the upper-class boundary against the cumulative frequency.

Amount Claimed (R1000s)	Upper Class Boundary	No. of Claims (frequency)	Cumulative Frequency	Points to Plot
	0	0	0	(0 ; 0)
$0 < x \leq 5$	5	150	150	(5 ; 150)
$5 < x \leq 10$	10	190	340	(10 ; 340)
$10 < x \leq 15$	15	250	590	(15 ; 590)
$15 < x \leq 20$	20	320	910	(20 ; 910)
$20 < x \leq 25$	25	260	1170	(25 ; 1170)
$25 < x \leq 30$	30	210	1380	(30 ; 1380)
$30 < x \leq 35$	35	140	1520	(35 ; 1520)
$35 < x \leq 40$	40	80	1600	(40 ; 1600)



Variance and Standard Deviation of Ungrouped Data

- Standard deviation is a measure of dispersion (spread of data) about the mean.
- Standard deviation = $\sqrt{\text{Variance}}$
 - Therefore, Variance is the $(\text{standard deviation})^2$

We use the following symbol for standard deviation $\rightarrow \sigma$

N.B Always use a calculator to calculate variance or standard deviation

Interpretation of Standard Deviation

- The bigger the standard deviation \rightarrow The more data is spread out
- The smaller the standard deviation \rightarrow The less data is spread out
- One Standard Deviation Interval $\rightarrow (\bar{x} - \sigma; \bar{x} + \sigma)$
- Two Standard Deviation Interval $\rightarrow (\bar{x} - 2\sigma; \bar{x} + 2\sigma)$

Example (Calculating mean and standard deviation (Ungrouped data))

Given data in the table below, determine the mean and standard deviation:

x	2	5	7
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Solution

In the following steps we used the calculator **CASIO fx-82ZA PLUS II**

Step 1	Press Mode button	
Step 2	Press number 2 for STAT	
Step 3	Choose 1-VAR by pressing number 1	
Step 4	In the X-column, enter all x-values, one after the other by pressing = after each entry.	
Step 5	Press AC button \rightarrow Press SHIFT button \rightarrow Press the number 1	
Step 6	Choose Var by pressing the number 4	
Step 7	Press the number 2 followed by = to get the mean and press the number 3 followed by = to get the standard deviation. N.B After getting each value, you need to repeat steps 5 – 6 to get the value in step 7. Press the AC button after getting each value.	Mean = 4,67 Standard Deviation = 2,05

N.B to get back to the normal (Comp.) mode, press **MODE button followed by number **1****

Example (Calculating mean and standard deviation (Grouped data))

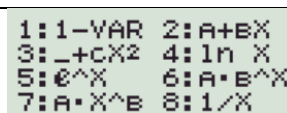
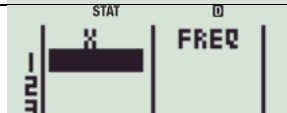
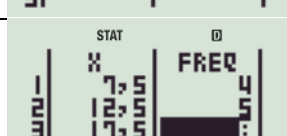
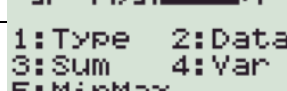
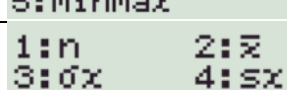
Given data in the table below, determine the estimated mean and standard deviation:

Class	Frequency
$5 < x \leq 10$	4
$10 < x \leq 15$	5
$15 < x \leq 20$	8

Solution

Class	Frequency	x_i (class midpoint)
$5 < x \leq 10$	4	7,5
$10 < x \leq 15$	5	12,5
$15 < x \leq 20$	8	17,5

In the following steps we used the calculator **CASIO fx-82ZA PLUS II**

Step 1	Since we are dealing with grouped data, we must first turn on the frequency feature as follows: Press SHIFT → MODE → Down Arrow (from REPLAY button) → Choose STAT by pressing 3 → B Choose ON by pressing 1 .	
Step 2	Press MODE → Choose STAT by pressing 2	
Step 3	Choose 1-VAR by pressing number 1	
Step 4	In the X-column, enter all class midpoint values, one after the other by pressing = after each entry. Then move to the Frequency column and enter corresponding frequencies.	
Step 5	Press AC button → Press SHIFT button → Press the number 1	
Step 6	Choose Var by pressing the number 4	
Step 7	Press the number 2 followed by = to get the estimate mean and press the number 3 followed by = to get the estimate standard deviation. N.B After getting each value, you need to repeat steps 5 – 6 to get the value in step 7. Press the AC button after getting each value.	Estimate Mean = 13,68 Estimate Standard Deviation = 4,03

N.B to turn off the frequency follow Step 1 (but choose 2 to turn OFF)

N.B to get back to the normal (Comp.) mode, press **MODE button followed by number 1**

Worked Example 1 (November 2008 Q9)**QUESTION 9**

The time taken, in minutes, to complete a 5 kilometre race by a group of 10 runners is given below:

18 21 16 24 28 20 22 29 19 23

- 9.1 Calculate the mean time taken to complete the race.
- 9.2 Calculate the standard deviation of the time taken to complete the race. (Use the formula on the information sheet.)
- 9.3 How many runners completed the race within one standard deviation of the mean?

Solution

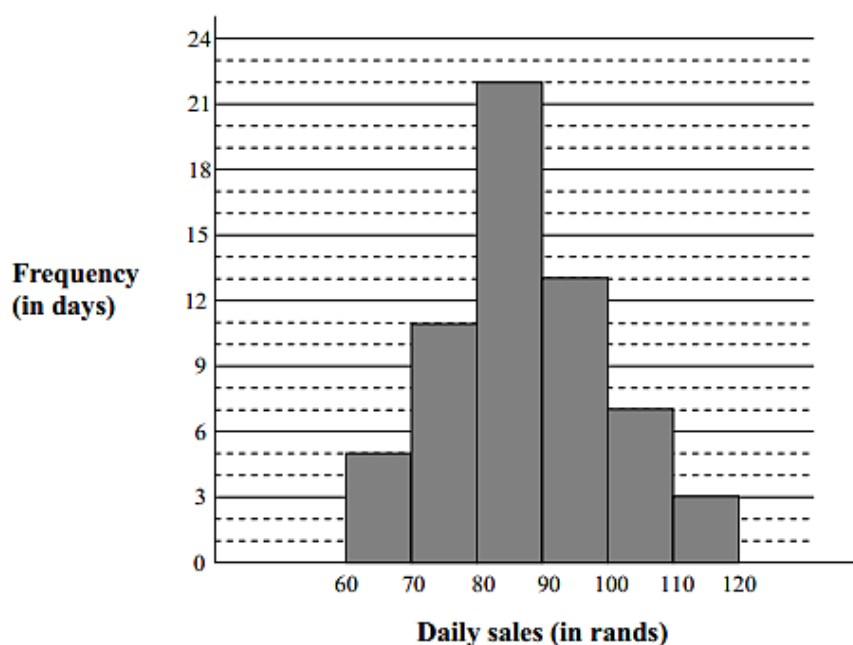
9.1	Mean = $\frac{220}{10} = 22$ minutes
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9.2 $\sigma = 3,95$

9.3	<p>One standard deviation of the mean is in the interval $(22 - 3,95 ; 22 + 3,95)$ which is $(18,05 ; 25,95)$</p> <p>\therefore 6 runners completed the race within one standard deviation of the mean. (List of times: 21, 24, 20, 22, 19, 23)</p>
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Worked Example 2 (November 2008 Q10)**QUESTION 10**

A street vendor has kept a record of sales for November and December 2007.
The daily sales in rands is shown in the histogram below.



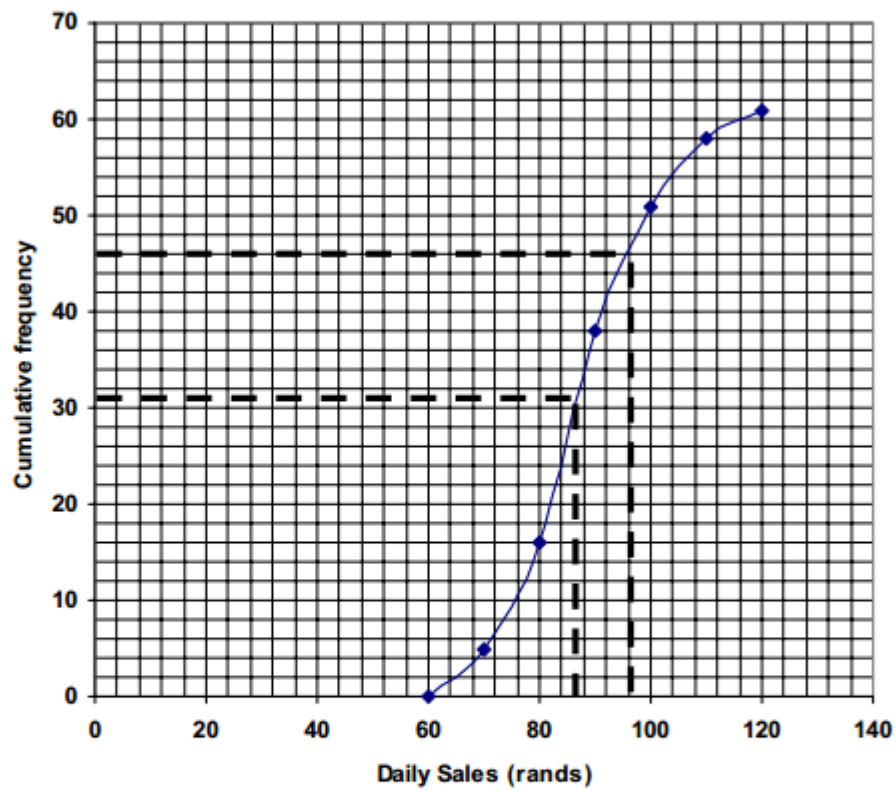
- 10.1 On DIAGRAM SHEET 3, complete the cumulative frequency table for the sales over November and December.
- 10.2 Draw an ogive for the sales over November and December on DIAGRAM SHEET 3.
- 10.3 Use your ogive to determine the median value for the daily sales. Explain how you obtain your answer.
- 10.4 Estimate the interval of the upper 25% of the daily sales.

Solution

10.1			
	Daily Sales (in Rand)	Frequency	Cumulative Frequency
	$60 \leq \text{rand} < 70$	5	5
	$70 \leq \text{rand} < 80$	11	16
	$80 \leq \text{rand} < 90$	22	38
	$90 \leq \text{rand} < 100$	13	51
	$100 \leq \text{rand} < 110$	7	58
	$110 \leq \text{rand} < 120$	3	61

10.2

Sales for November and December 2007



10.3 Median = R 87
(Accept answers between 84 and 90)

10.4 $R\ 96 \leq \text{sales} \leq R\ 120$

Regression Line (Line of Best Fit or The Least Squares Line)

Formula to find the predicted y –value (\hat{y}): $\hat{y} = a + bx$

N.B Use a calculator to find the value of a and b .

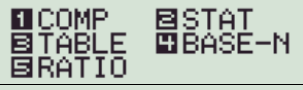
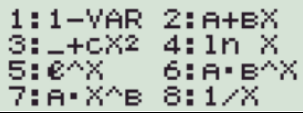


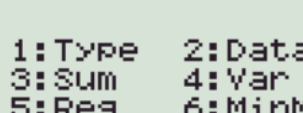
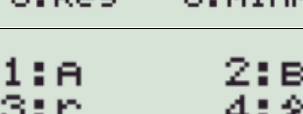
Example (Finding the equation of regression line)

Given data in the table below, determine the equation of line of best fit:

x	2	5	7
y	5	9	13

Solution

In the following steps we used the calculator **CASIO fx-82ZA PLUS II**

Step 1	Press Mode button	
Step 2	Press number 2 for STAT	
Step 3	Choose option 2: A+BX by pressing the number 2	
Step 4	In the X-column, enter all x-values, one after the other by pressing = after each entry. Then move to the Y-column, enter all y-values, one after the other by pressing = after each entry.	
Step 5	Press AC button → Press SHIFT button → Press the number 1	
Step 6	Choose 5:Reg by pressing the number 5	
Step 7	Press the number 1 followed by = to get the value of A , and press the number 2 followed by = to get the value of B . You can also get the value of r (correlation coefficient). N.B After getting each value, you need to repeat steps 5 – 6 to get the value in step 7. Press the AC button after getting each value.	$\begin{aligned} A &= 1,63 \\ B &= 1,58 \\ r &= 0,99 \end{aligned}$

The equation of line of best fit is then $\hat{y} = 1,63 + 1,58x$

N.B to get back to the normal (Comp.) mode, press **MODE button followed by number **1****

Drawing the line of best fit

Approach 1

Substitute any two x –values from the table to get the predicted y –values.

Then plot the two points and join them.

N.B Ensure that your line is long enough to cover the

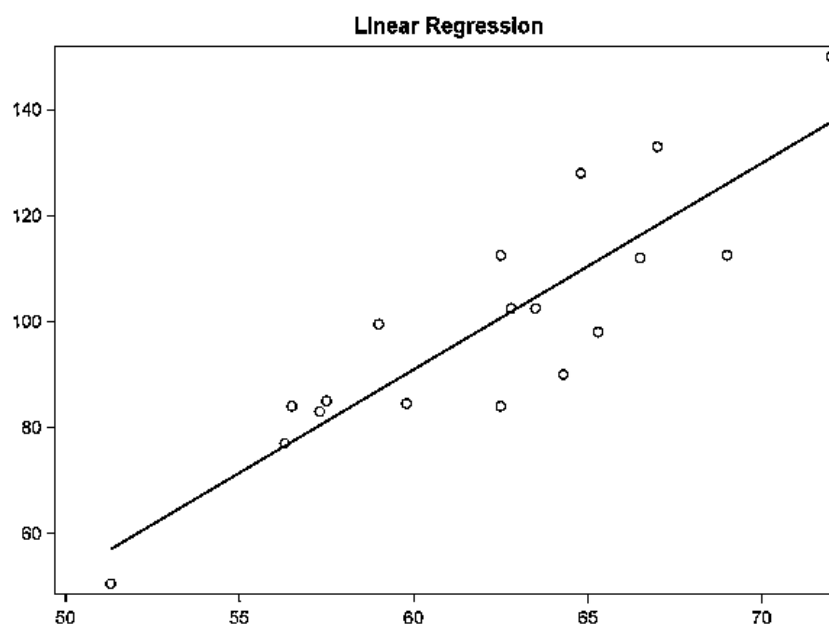
Approach 2

Determine the mean point and the y -intercept, then join the two points.

To determine the mean point, use the example on the previous page, follow steps 1 – 5, then the following steps:

Step 6	Choose 4:Var by pressing the number 4	<div> <div>1:n</div> <div>3:σx</div> <div>5:ȳ</div> <div>7:Sy</div> </div> <div> <div>2:Σ</div> <div>4:Σx</div> <div>6:σy</div> </div>
Step 7	Press the number 2 followed by = to get the mean for x , and press the number 5 followed by = to get the mean for y . N.B After getting each value, you need to repeat steps 5 – 6 to get the value in step 7. Press the AC button after getting each value.	$\bar{x} = 4,67$ $\bar{y} = 9$ \therefore mean point is (4,67 ; 9)

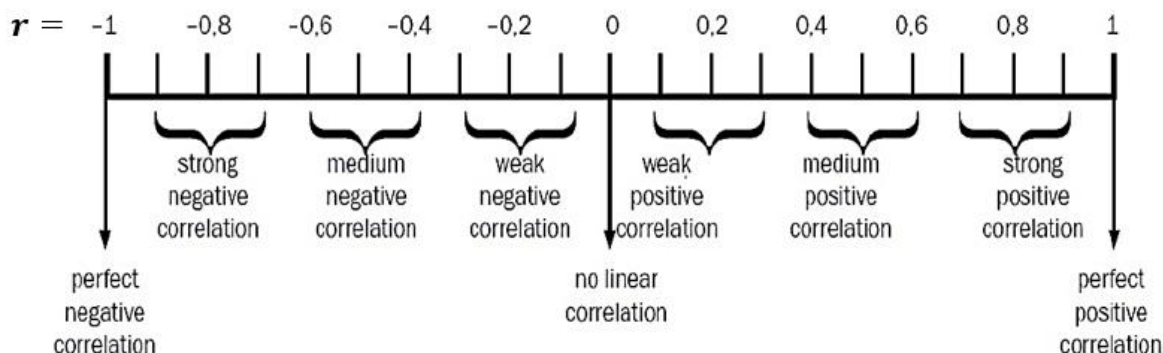
N.B When drawing the line of best fit, ensure that the line is long enough to cover the grid, see below:



Correlation Coefficient

- Correlation coefficient (r) indicates the strength of the relationship between the two variables. Use calculator to calculate r (See example on page 80).
- $-1 \leq r \leq 1$

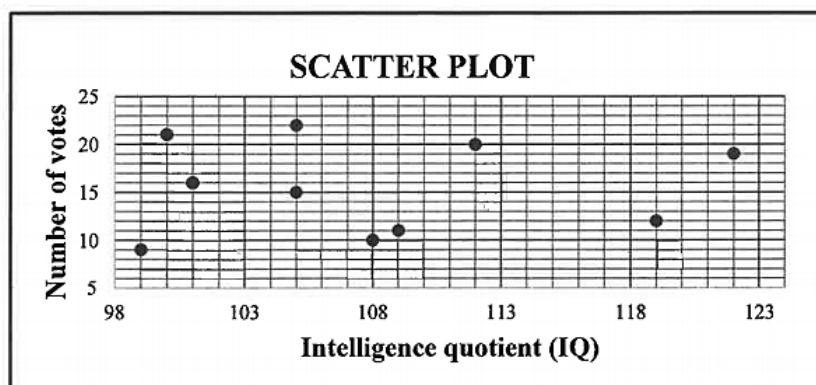
Reading/Describing Correlation Coefficient



Worked Example (November 2022 Q1)

QUESTION 1

The matric class of a certain high school had to vote for the chairperson of the RCL (representative council of learners). The scatter plot below shows the IQ (intelligence quotient) of the 10 learners who received the most votes and the number of votes that they received.



Before the election, the popularity of each of these ten learners was established and a popularity score (out of a 100) was assigned to each. The popularity scores and the number of votes of the same 10 learners who received the most votes are shown in the table below.

Popularity score (x)	32	89	35	82	50	59	81	40	79	65
Number of votes (y)	9	22	10	21	11	15	20	12	19	16

- 1.1 Calculate the:
- 1.1.1 Mean number of votes that these 10 learners received
- 1.1.2 Standard deviation of the number of votes that these 10 learners received
- 1.2 The learners who received fewer votes than one standard deviation below the mean were not invited for an interview. How many learners were invited?
- 1.3 Determine the equation of the least squares regression line for the data given in the table.
- 1.4 Predict the number of votes that a learner with a popularity score of 72 will receive.
- 1.5 Using the scatter plot and table above, provide a reason why:
- 1.5.1 IQ is not a good indicator of the number of votes that a learner could receive
- 1.5.2 The prediction in QUESTION 1.4 is reliable

Solution

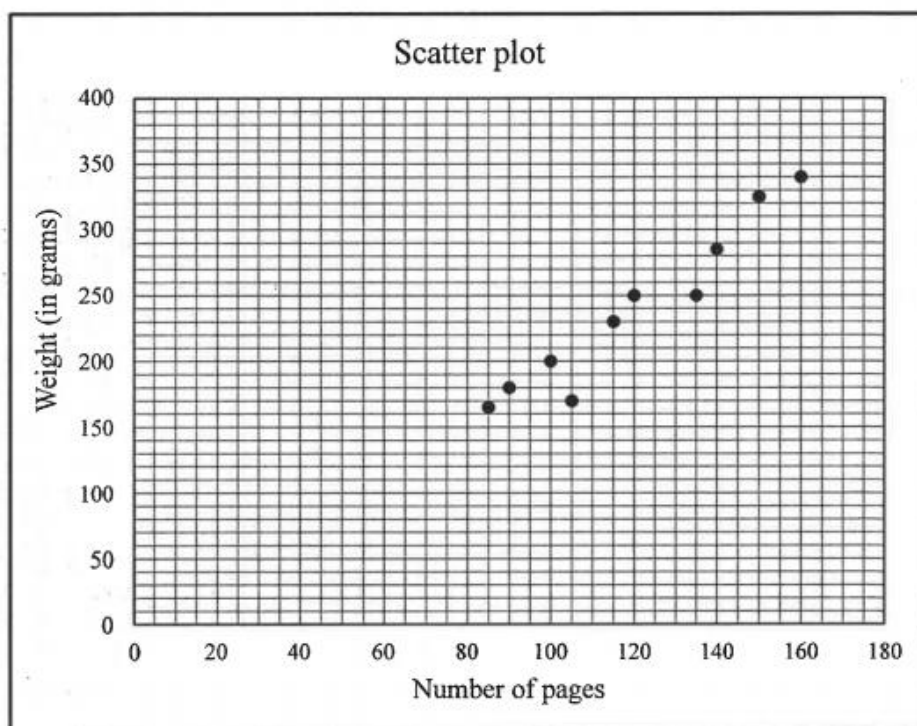
1.1.1	$\bar{y} = \frac{155}{10}$ $= 15,5$
1.1.2	SD = 4,59
1.2	$\bar{y} - \text{SD}$ $= 15,5 - 4,59$ $= 10,91$ $\therefore 10 - 2 = 8 \text{ learners}$
1.3	$a = 1,7709\dots$ $b = 0,2243\dots$ $\hat{y} = 1,77 + 0,22x$
1.4	$\hat{y} = 1,77 + 0,22(72)$ $= 17,61$ $\approx 18 \text{ votes}$ <p>OR/OF</p> $\hat{y} = 17,92 \approx 18 \text{ votes}$

1.5.1	Points are all scattered therefore low correlation and unrealistic prediction./Punte is versprei daarom 'n lae korrelasie en onrealistiese voorspelling.
1.5.2	$r = 0,98$ /correlation very strong/korrelasie baie sterk \therefore a reliable prediction/'n betroubare voorspelling

ACTIVITIES**Statistics and Regression***(May/June 2024)***QUESTION 1**

The number of pages in ten A4 books and their corresponding weights (in grams) are given in the table below. The data is also represented in the scatter plot.

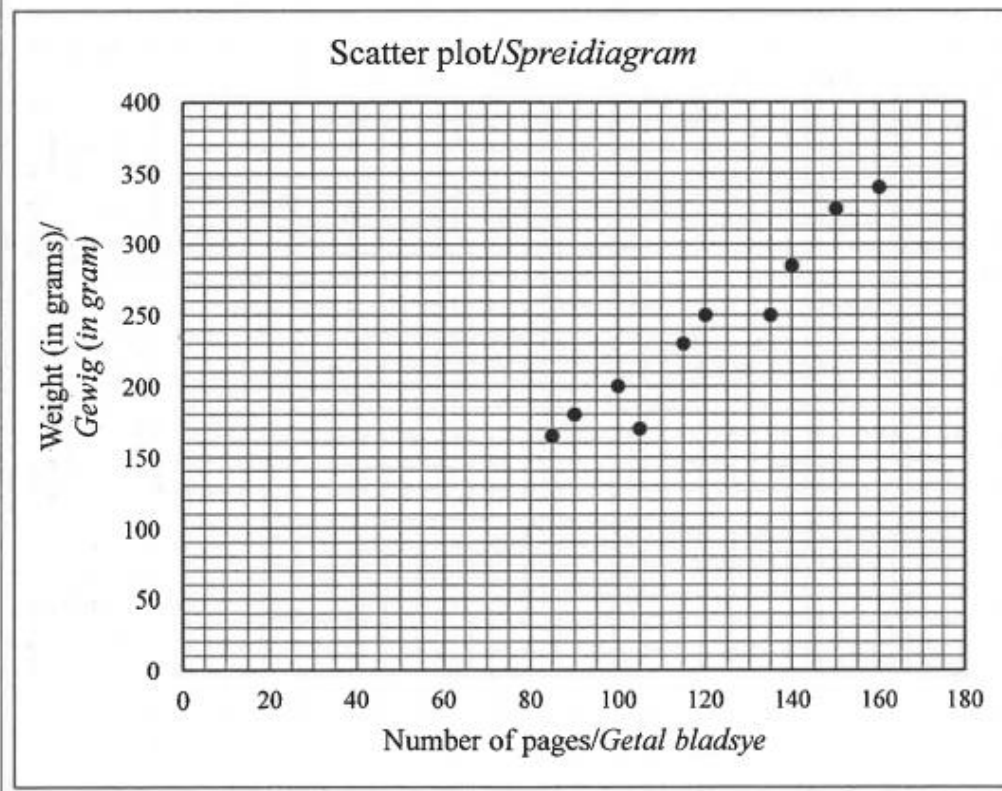
Number of pages (x)	85	150	100	120	90	140	135	105	115	160
Weight (in grams) (y)	165	325	200	250	180	285	250	170	230	340



- 1.1 Determine the equation of the least squares regression line. (3)
 - 1.2 Draw the least squares regression line on the scatter plot in the ANSWER BOOK. (2)
 - 1.3 Predict the weight of an A4 book that has 110 pages. (2)
 - 1.4 Calculate the percentage weight increase between a book with 110 pages and a book with 130 pages. (3)
- [10]**

ANSWER QUESTION 1.2 HERE:

1.2



QUESTION 2

Fifty athletes need to access suitable training facilities. The table below shows the distances, in km, that they need to travel to obtain access to suitable training facilities.

DISTANCE (x km)	NUMBER OF ATHLETES
$0 \leq x < 5$	3
$5 \leq x < 10$	7
$10 \leq x < 15$	20
$15 \leq x < 20$	12
$20 \leq x < 25$	5
$25 \leq x < 30$	3

- 2.1 Complete the cumulative frequency column provided in the table in the ANSWER BOOK. (2)
- 2.2 On the grid provided in the ANSWER BOOK, draw a cumulative frequency graph (ogive) to represent the above data. (3)
- 2.3 Calculate the interquartile range (IQR) of the above data. (2)
- 2.4 The families of 4 of the athletes above who stay between 15 and 20 km from a suitable training facility, decide to move 10 kilometres closer to the facility. In which interval will the number of athletes increase? (1)
- 2.5 Calculate the estimated mean distance that the fifty athletes need to travel after the 4 families have moved 10 kilometres closer to the facility. Clearly show ALL working. (3)
- [11].**

ANSWER QUESTION 2.1 AND 2.2 HERE:

2.1

Distance/ <i>Afstand</i> (x km)	Number of athletes/ <i>Getal atlete</i>	Cumulative frequency/ <i>Kumulatiewe frekwensie</i>
$0 \leq x < 5$	3	
$5 \leq x < 10$	7	
$10 \leq x < 15$	20	
$15 \leq x < 20$	12	
$20 \leq x < 25$	5	
$25 \leq x < 30$	3	

2.2

Ogive/Ogief

The graph is a coordinate plane with a grid. The vertical axis (y-axis) is labeled 'Cumulative frequency / Kumulatiewe frekwensie' and has major tick marks every 5 units from 0 to 55. The horizontal axis (x-axis) is labeled 'Distance / Afstand (in km)' and has major tick marks every 5 units from 0 to 35. The grid consists of small squares, with each major tick mark on the y-axis corresponding to 10 small squares and each major tick mark on the x-axis corresponding to 10 small squares.

*(May/June 2023)***QUESTION 1**

- 1.1 The owner of a small company wishes to establish whether advertising in a regional newspaper is effective. The table below shows the amount spent on advertising and the corresponding sales figures for the last 9 years.

Amount spent on advertising (in rands) (x)	21 300	23 700	24 800	30 540	24 100	40 680	22 400	35 250	29 110
Sales (in rands) (y)	311 500	326 700	349 200	470 000	316 100	564 200	314 000	487 300	392 900

- 1.1.1 Determine the equation of the least squares regression line for the data. (3)
- 1.1.2 Predict the sales for a year in which the company will spend R28 500 on advertising. (2)
- 1.1.3 Write down the correlation coefficient of the data. (1)
- 1.1.4 Describe the association between the amount spent on advertising in the regional newspaper and the sales of this company. (1)
- 1.2 The profit that the small company made over the same 9 years is given in the table below.

Profit (in rands)	110 750	107 376	152 338	244 480	144 021	275 994	121 900	207 636	187 700
--------------------------	---------	---------	---------	---------	---------	---------	---------	---------	---------

- 1.2.1 Calculate the mean profit made over the 9 years. (2)
- 1.2.2 Write down the standard deviation for the data. (1)
- 1.2.3 Determine the number of years in which the company made a profit that was greater than one standard deviation above the mean. (2)
- [12]**

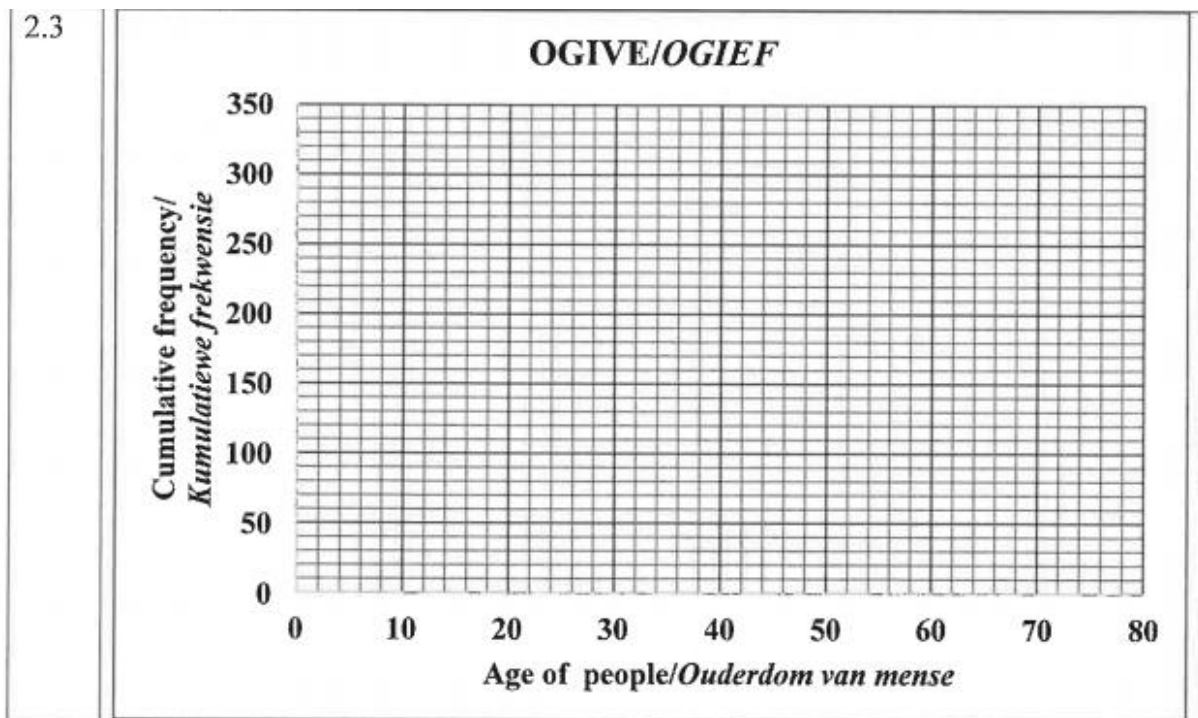
QUESTION 2

The ages of the people who attended a music concert was summarised in the table below.

AGE	NUMBER OF PEOPLE
$5 < x \leq 15$	20
$15 < x \leq 25$	25
$25 < x \leq 35$	60
$35 < x \leq 45$	90
$45 < x \leq 55$	55
$55 < x \leq 65$	40
$65 < x \leq 75$	30

- 2.1 Write down the modal class of the data. (1)
- 2.2 How many people attended the music concert? (1)
- 2.3 On the grid provided in the ANSWER BOOK, draw a cumulative frequency graph (ogive) to represent the above data. (4)
- 2.4 Use the cumulative frequency graph to determine the median age of the people who attended the music concert. (2)
- [8]**

ANSWER QUESTION 2.3 HERE:



*(May/June 2022)***QUESTION 1**

The table below shows the mass (in kg) of the school bags of 80 learners.

MASS (in kg)	FREQUENCY
$5 < m \leq 7$	6
$7 < m \leq 9$	18
$9 < m \leq 11$	21
$11 < m \leq 13$	19
$13 < m \leq 15$	11
$15 < m \leq 17$	4
$17 < m \leq 19$	1

- 1.1 Write down the modal class of the data. (1)
- 1.2 Complete the cumulative frequency column in the table in the ANSWER BOOK. (2)
- 1.3 Draw a cumulative frequency graph (ogive) for the given data on the grid provided in the ANSWER BOOK. (3)
- 1.4 Use the graph to determine the median mass for this data. (2)
- 1.5 The international guideline for the mass of a school bag is that it should not exceed 10% of a learner's body mass.
 - 1.5.1 Calculate the estimated mean mass of the school bags. (2)
 - 1.5.2 The mean mass of this group of learners was found to be 80 kg. On average, are these school bags satisfying the international guideline with regard to mass? Motivate your answer. (2)

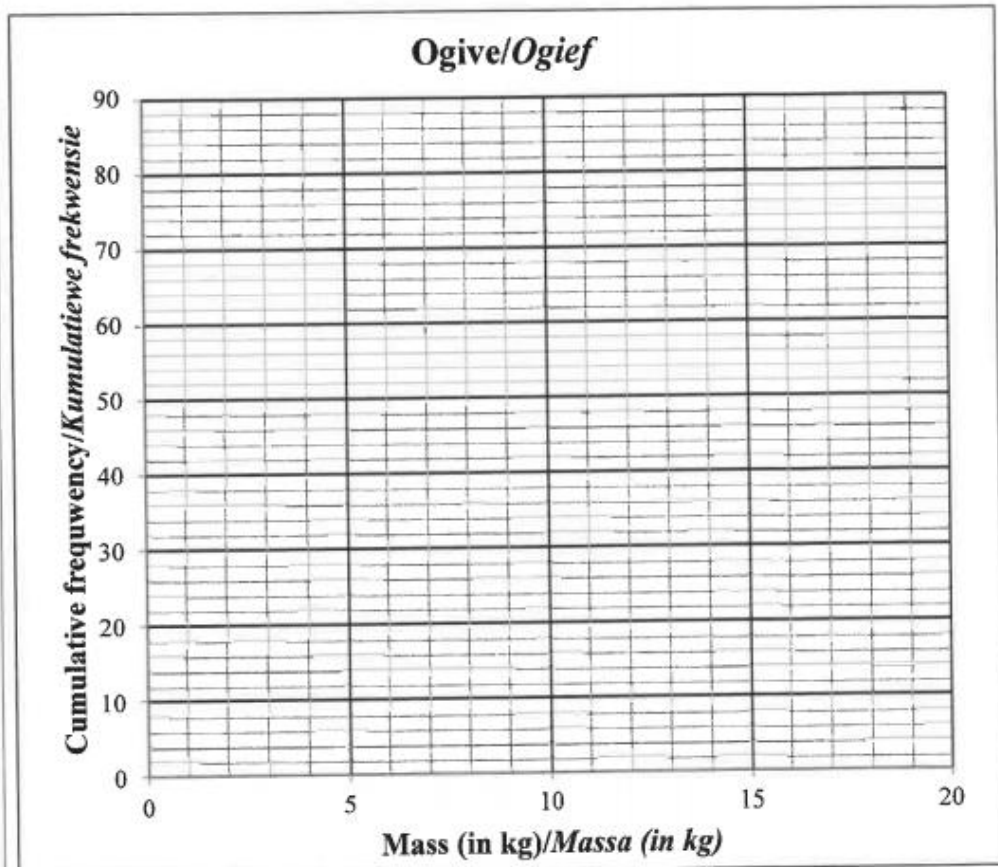
[12]

ANSWER QUESTION 1.2 AND 1.3 HERE:

1.2

MASS (in kg)/ MASSA (in kg)	FREQUENCY/ FREKWENSIE	CUMULATIVE FREQUENCY/ KUMULATIEWE FREKWENSIE
$5 < m \leq 7$	6	
$7 < m \leq 9$	18	
$9 < m \leq 11$	21	
$11 < m \leq 13$	19	
$13 < m \leq 15$	11	
$15 < m \leq 17$	4	
$17 < m \leq 19$	1	

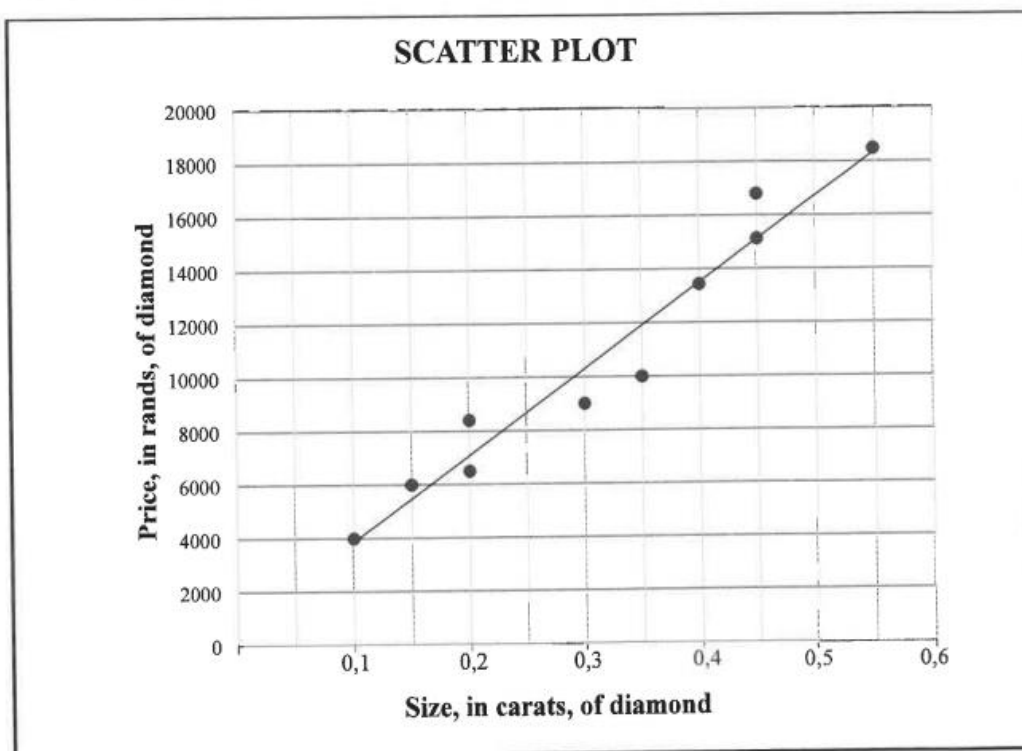
1.3



QUESTION 2

The table below shows the size (in carats) and the price (in rands) of 10 diamonds that were sold by a diamond trader. This information is also presented in the scatter plot below. The least squares regression line for the data is drawn.

Size, in carats, of diamond (x)	0,1	0,15	0,2	0,2	0,3	0,35	0,4	0,45	0,45	0,55
Price, in rands, of diamond (y)	4 000	6 000	6 500	8 400	9 000	10 000	13 440	15 120	16 800	18 480



- 2.1 Determine the equation of the least squares regression line for the data. (3)
- 2.2 If the trader sold a diamond that was 0,25 carats in size, predict the selling price of this diamond in rands. (2)
- 2.3 Calculate the average price increase per 0,05 carat of the diamonds. (2)
- 2.4 It was later found that the selling price of the 0,35 carat diamond was recorded incorrectly. The correct price is R11 500. When this correction is made to the data set, the correlation between the size and price of these diamonds gets stronger. Explain the reason for this by referring to the given scatter plot. (1)

[8]

ANALYTICAL GEOMETRY

Straight Lines

① Distance formula:

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

② Midpoint formula:

$$M \left(\frac{x_1 + x_2}{2} ; \frac{y_1 + y_2}{2} \right)$$

③ The gradient / slope of a line:

$$m = \frac{y_2 - y_1}{x_2 - x_1} \quad \text{or} \quad \frac{y_1 - y_2}{x_1 - x_2}$$

order is very important!
You HAVE to use the
same order in the numerator
and denominator.

↖ This line is increasing.
It has a positive gradient.

↘ This line is decreasing.
It has a negative gradient.

④ The equation of a straight line:

A straight line equation is written in the form $y = mx + c$.

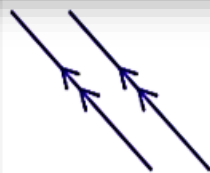
$$y = mx + c$$

↖ gradient ↘ y-intercept

or

$$y - y_1 = m(x - x_1)$$

↖ gradient
↘ a point on the line
(x_1, y_1)



Parallel Lines



Perpendicular
Lines

Parallel lines

-> same gradient

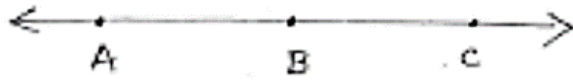
$$\therefore m_1 = m_2$$

Perpendicular lines

-> Product of
gradients = -1

$$\therefore m_1 \times m_2 = -1$$

Collinear points → Two or more points are called collinear if all points lie on the same straight line.

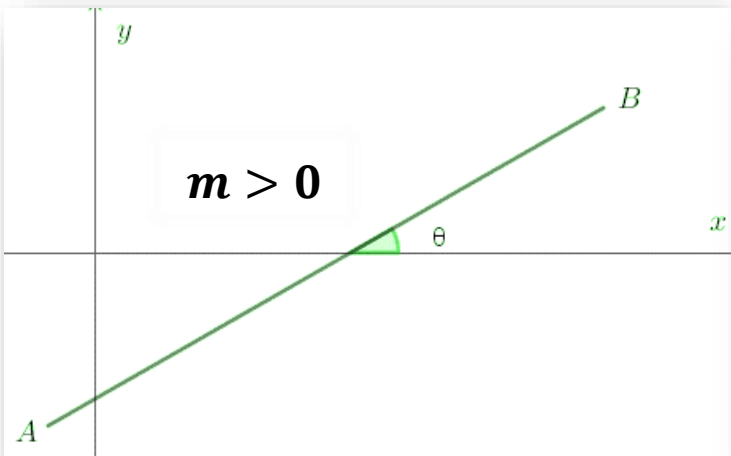


In this line, A, B, C are collinear points

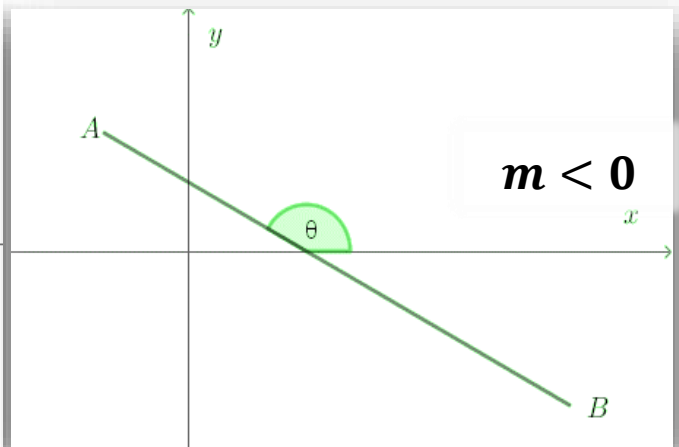
$$\therefore m_{AB} = m_{BC} = m_{AC}$$

Inclination of a line

$$m = \tan \theta$$

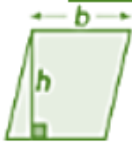







$$\theta = \tan^{-1}(m)$$



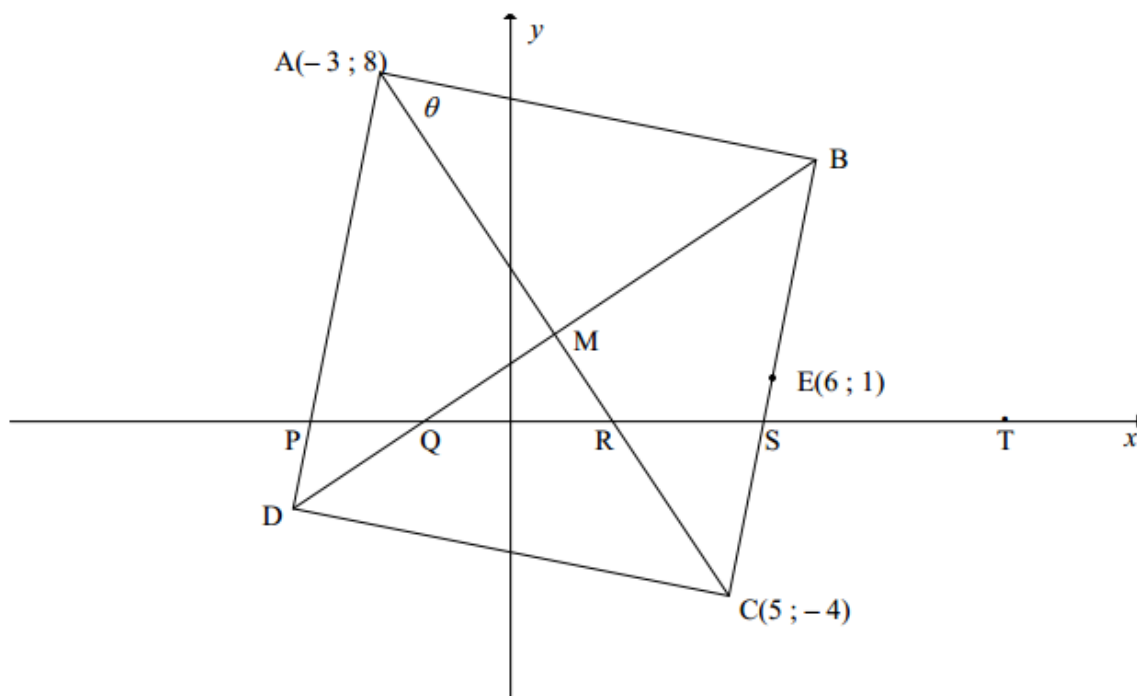
$$\theta = \tan^{-1}(m) + 180^\circ$$

Properties of Quadrilaterals

<u>Quadrilateral</u>	<u>Shape</u>	<u>Properties</u>	<u>Area</u>
Parallelogram		<ul style="list-style-type: none"> • Opposite sides parallel • Opposite sides equal • Opposite angles equal • Diagonals bisect each other 	$b \times h$
Rectangle		<ul style="list-style-type: none"> • All properties of parallelogram • All angles are right angles • Diagonals are equal in length 	$l \times b$
Square		<ul style="list-style-type: none"> • All properties of rectangle • All sides are equal • Diagonals bisect at right angle • Diagonals bisect corner angles 	x^2
Rhombus		<ul style="list-style-type: none"> • All properties of a parallelogram • All sides are equal • Diagonals bisect at right angle • Diagonals bisect corner angles 	$\frac{1}{2} \times d_1 \times d_2$
Kite		<ul style="list-style-type: none"> • Two pairs of adjacent sides are equal • One pair of opposite angles are equal • One diagonal bisect the other at right angle • One diagonal bisects corner angles 	$\frac{1}{2} \times d_1 \times d_2$
Trapezium		One pair of opposite sides parallel	$\frac{1}{2} \times (a + b) \times h$

Worked Example 1 (November 2012)**QUESTION 5**

ABCD is a rhombus with $A(-3; 8)$ and $C(5; -4)$. The diagonals of ABCD bisect each other at M. The point $E(6; 1)$ lies on BC.



- 5.1 Calculate the coordinates of M.
- 5.2 Calculate the gradient of BC.
- 5.3 Determine the equation of the line AD in the form $y = mx + c$.
- 5.4 Determine the size of θ , that is \hat{BAC} . Show ALL calculations.

Solution

5.1	<p>Diagonals bisect each other at M:</p> $x_M = \frac{-3 + 5}{2} = 1 \quad ; \quad y_M = \frac{8 + (-4)}{2} = 2$ <p>M(1 ; 2)</p>
5.2	$m_{BC} = \frac{1 + 4}{6 - 5}$ $m_{BC} = 5$ <p>OR</p> $m_{BC} = \frac{-4 - 1}{5 - 6}$ $m_{BC} = 5$
5.3	$y - y_1 = m(x - x_1)$ $y - 8 = m(x + 3)$ $m_{AD} = m_{BC} = 5$ $y - 8 = 5(x + 3)$ $y = 5x + 23$ <p>Lines parallel</p>

5.4

ABCD is a rhombus, therefore

$$AB = BC$$

$$\theta = \hat{BCA} = \hat{ARS} - \hat{RSC}$$

$$= \hat{ARS} - \hat{BST}$$

$$\tan \hat{ARS} = m_{AC} = \frac{8+4}{-3-5}$$

$$\tan \hat{ARS} = -\frac{3}{2}$$

$$\hat{ARS} = 180^\circ - 56,3099\dots$$

$$\hat{ARS} = 123,69^\circ$$

$$\tan \hat{BST} = m_{BC} = 5$$

$$\hat{BST} = 78,69^\circ$$

$$\theta = \hat{BCA} = 123,69^\circ - 78,69^\circ$$

$$\theta = 45^\circ$$

OR

$$\tan \hat{ARS} = m_{AC} = \frac{8+4}{-3-5} = -\frac{3}{2}$$

$$\hat{ARS} = 123,69^\circ$$

$$\tan \hat{APR} = m_{AD} = 5$$

$$\hat{APR} = 78,69^\circ$$

$$\hat{PAR} = \hat{ARS} - \hat{APR} \quad \text{Exterior angle of a triangle}$$

$$= 123,69^\circ - 78,69^\circ$$

$$= 45^\circ$$

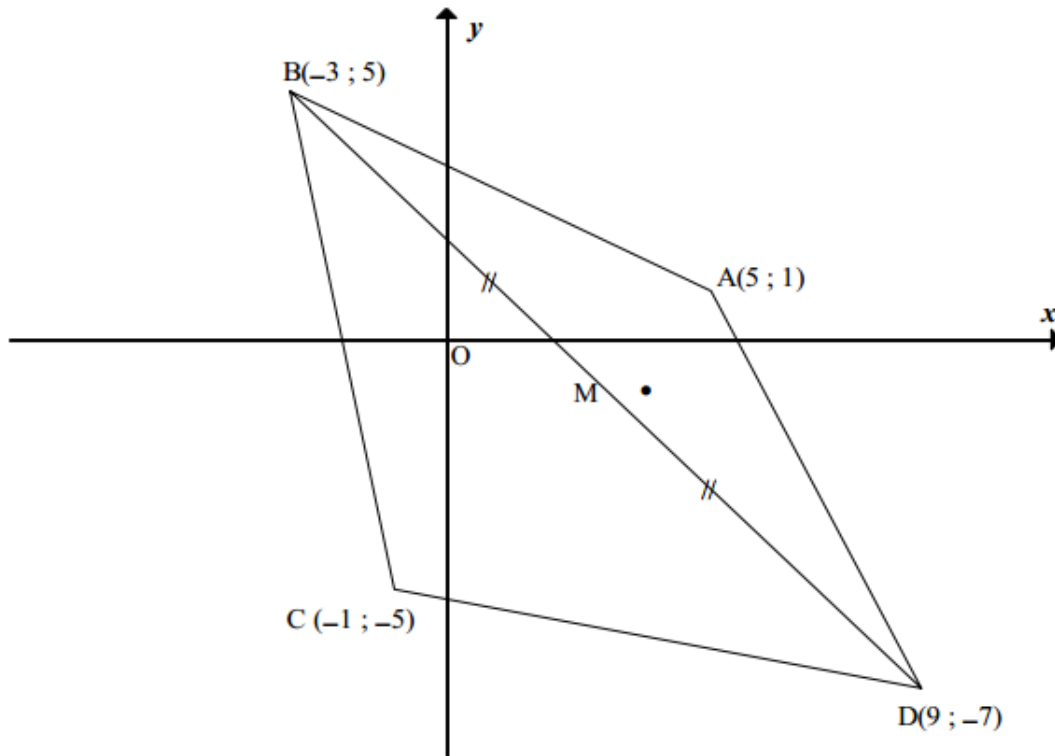
$$\theta = \hat{PAR}$$

$$= 45^\circ$$

Diagonals of the rhombus bisect
opposite angles

Worked Example 2 (November 2009)**QUESTION 1**

ABCD is a quadrilateral with vertices A(5 ; 1), B(-3 ; 5), C(-1 ; -5) and D(9 ; -7).



- 1.1 Calculate the gradient of AC.
- 1.2 Determine the equation of AC in the form $y = \dots$
- 1.3 Hence, or otherwise, show that the midpoint M of BD lies on AC.
- 1.4 Show that $\angle AMB = 90^\circ$.
- 1.5 Calculate the area of $\triangle ABC$.

Solution

1.1	$m_{AC} = \frac{1 - (-5)}{5 - (-1)}$ $m_{AC} = 1$
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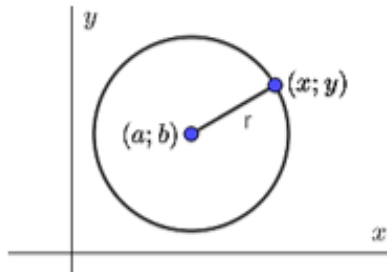
1.2	Equation of AC is: $y - 1 = 1(x - 5)$ $y = x - 4$
1.3	$M(x; y) = \left(\frac{9 - 3}{2}; \frac{-7 + 5}{2} \right)$ $= (3; -1)$ Substitute $x = 3$ into equation of AC: $y = 3 - 4 = -1$ $\therefore M$ lies on line AC
1.4	$m_{BM} = \frac{5 + 1}{-3 - 3}$ $m_{BM} = -1$ $m_{AC} = 1$ $m_{BM} \cdot m_{AC} = -1 \times 1$ $= -1$ $\therefore BM \perp AC$ or $\hat{AMB} = 90^\circ$
1.5	$BM = \sqrt{(5 + 1)^2 + (-3 - 3)^2}$ $BM = \sqrt{72}$ $AC = \sqrt{(5 + 1)^2 + (1 + 5)^2}$ $AC = \sqrt{72}$ Area of $\triangle ABC = \frac{1}{2}(\sqrt{72})(\sqrt{72})$ $= 36$ square units

Equation of A Circle**Standard Form:**

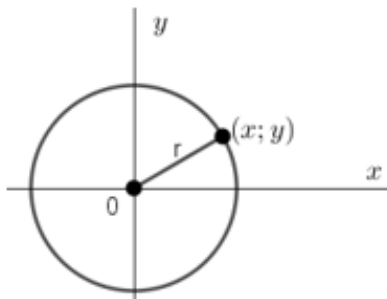
$$(x - a)^2 + (y - b)^2 = r^2$$

centre = $(a; b)$

radius = $\sqrt{r^2} = r$



centre at the origin $(0; 0)$: $x^2 + y^2 = r^2$

**General form:**

$$x^2 + y^2 + cx + dy + e = 0$$

Given the equation of the circle $x^2 + y^2 - 6x + 8y + 7 = 0$

Determine the centre and the radius:

$$x^2 - 6x + y^2 + 8y = -7$$

$$x^2 - 6x + (\quad)^2 + y^2 + 8y + (\quad)^2 = -7 + (\quad)^2 + (\quad)^2$$

$$x^2 - 6x + (-3)^2 + y^2 + 8y + (4)^2 = -7 + (-3)^2 + (4)^2$$

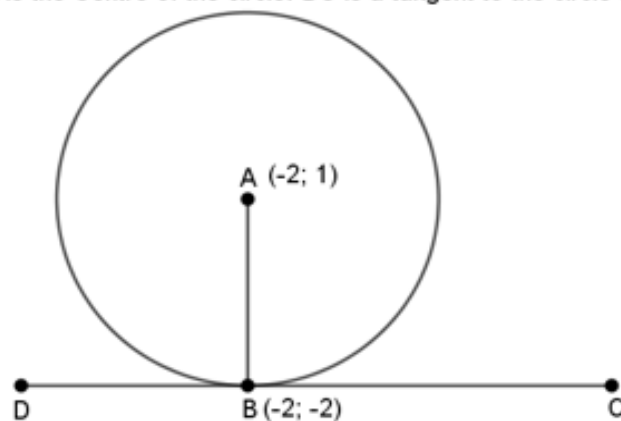
$$(x - 3)^2 + (y + 4)^2 = 18$$

centre = $(3; -4)$

radius = $3\sqrt{2}$

Determining the equation of a tangent to the circle

A is the Centre of the circle. DC is a tangent to the circle at B.



1. Determine the gradient of the radius AB

2. $AB \perp DC$ (tangent \perp radius)

3. Determine the gradient of tangent DC

$$m_1 \times m_2 = -1$$

4. Then find the equation of the tangent DC

$$y = mx + c$$

Worked Example 1 (Feb. – March 2010)**QUESTION 6**

- 6.1 Determine the centre and radius of the circle with the equation $x^2 + y^2 + 8x + 4y - 38 = 0$.
- 6.2 A second circle has the equation $(x - 4)^2 + (y - 6)^2 = 26$. Calculate the distance between the centres of the two circles.
- 6.3 Hence, show that the circles described in QUESTION 6.1 and QUESTION 6.2 intersect each other.
- 6.4 Show that the two circles intersect along the line $y = -x + 4$.

Solution

6.1	$x^2 + y^2 + 8x + 4y - 38 = 0$ $x^2 + 8x + 16 + y^2 + 4y + 4 = 16 + 4 + 38$ $(x + 4)^2 + (y + 2)^2 = 58$ <p>Centre is $(-4 ; -2)$ and the radius is $\sqrt{58}$</p>
6.2	<p>Centre of second circle is $(4 ; 6)$</p> <p>Distance between centres is $\sqrt{(4 + 4)^2 + (6 + 2)^2} = \sqrt{128} = 11,31$</p>
6.3	<p>Sum of radii = $\sqrt{58} + \sqrt{26} = 12,71$</p> <p>Distance between centres is 11,31.</p> <p>sum of the radii > distance between the centres</p> <p>\therefore the circles must overlap and hence the circles must intersect.</p>

6.4

Equation of second circle:

$$(x-4)^2 + (y-6)^2 = 26$$

$$x^2 - 8x + 16 + y^2 - 12y + 36 = 26$$

$$x^2 - 8x + y^2 - 12y + 26 = 0$$

Let $(x; y)$ be either of the two points on intersection.

Then

$$x^2 + y^2 + 8x + 4y - 38 = 0$$

$$\text{and } x^2 + y^2 - 8x - 12y + 26 = 0$$

$$\begin{array}{r} \text{Subtract} \quad \hline 16y + 16x - 64 = 0 \\ y = -x + 4 \end{array}$$

Both points of intersection lie on this line.

 $\therefore y = -x + 4$ is the equation of the common chord.**OR**Check that the line $y = -x + 4$ cuts the two circles at the same points:

$$(x-4)^2 + (-x-2)^2 = 26$$

$$x^2 - 8x + 16 + x^2 + 4x + 4 = 26$$

$$2x^2 - 4x - 6 = 0$$

$$x^2 - 2x - 3 = 0$$

$$(x-3)(x+1) = 0$$

$$x = 3 \text{ or } x = -1$$

$$x^2 + y^2 + 8x + 4y - 38 = 0$$

$$x^2 + (4-x)^2 + 8x + 4(4-x) - 38 = 0$$

$$x^2 + 16 - 8x + x^2 + 8x + 16 - 4x - 38 = 0$$

$$2x^2 - 4x - 6 = 0$$

$$x^2 - 2x - 3 = 0$$

$$x = 3 \text{ or } x = -1$$

Worked Example 2 (Feb. – March 2011)**QUESTION 5**

- 5.1 The equation of a circle is $x^2 + y^2 - 8x + 6y = 15$.
- 5.1.1 Prove that the point $(2 ; -9)$ is on the circumference of the circle.
- 5.1.2 Determine an equation of the tangent to the circle at the point $(2 ; -9)$.
- 5.2 Calculate the length of the tangent AB drawn from the point $A(6 ; 4)$ to the circle with equation $(x - 3)^2 + (y + 1)^2 = 10$.

Solution

5.1.1	$x^2 + y^2 - 8x + 6y$ $= (2)^2 + (-9)^2 - 8(2) + 6(-9)$ $= 4 + 81 - 16 - 54$ $= 15$ <p>Hence, the point lies on the circumference of the circle.</p> <p>OR</p> $x^2 + y^2 - 8x + 6y = 15$ $(x - 4)^2 + (y + 3)^2 = 15 + 16 + 9$ $(x - 4)^2 + (y + 3)^2 = 40$ $(x - 4)^2 + (y + 3)^2$ $= (2 - 4)^2 + (-9 + 3)^2$ $= 2^2 + 6^2$ $= 40$ <p>\therefore The point lies on the circumference of the circle.</p>
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5.1.2

$$x^2 + y^2 - 8x + 6y = 15$$

$$(x-4)^2 + (y+3)^2 = 15 + 16 + 9$$

$$(x-4)^2 + (y+3)^2 = 40$$

Circle centre (4 ; -3)

$$m_{rad} = \frac{-3 - (-9)}{4 - 2}$$

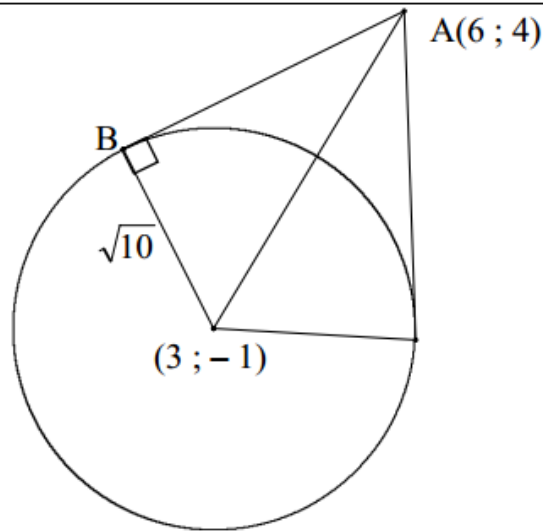
$$m_{rad} = 3$$

$$m_{tan} = -\frac{1}{3}$$

$$y + 9 = -\frac{1}{3}(x - 2)$$

$$y = -\frac{1}{3}x - \frac{25}{3}$$

5.2



$$\text{Radius } AB = \sqrt{10}$$

Distance from A to centre of circle is

$$= \sqrt{(6-3)^2 + (4+1)^2}$$

$$= \sqrt{9 + 25}$$

$$= \sqrt{34}$$

$$AB^2 = 34 - 10$$

$$AB^2 = 24$$

$$AB = \sqrt{24}$$

$$AB = 2\sqrt{6}$$

$$AB = 4,90$$

OR

$$r^2 = 10$$

$$r = \sqrt{10}$$

Radius \perp tangent

By Pythagoras

$$AB^2 = (6-3)^2 + (4+1)^2 - 10$$

$$= 24$$

$$AB = 4,90$$

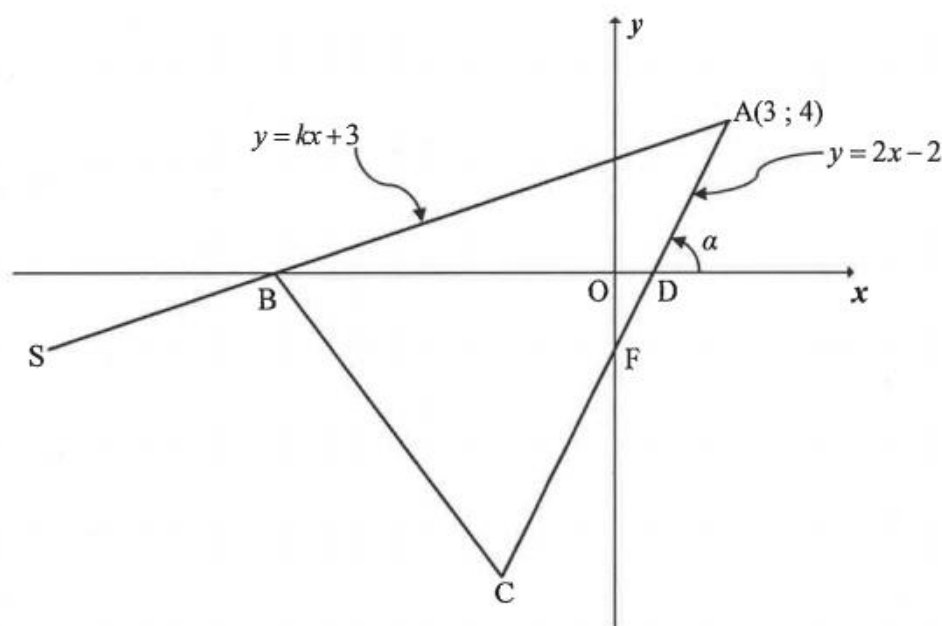
ACTIVITIES

Analytical Geometry

(May/June 2024)

QUESTION 3

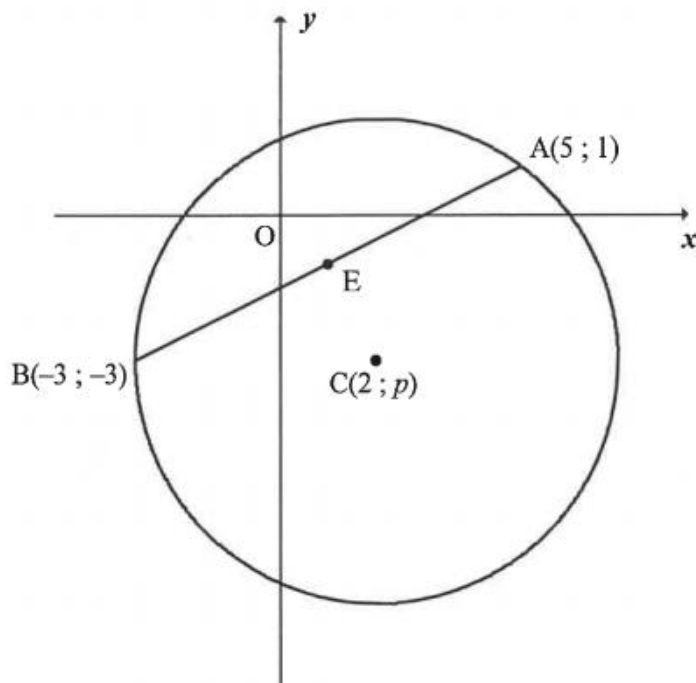
In the diagram, $A(3; 4)$, B and C are vertices of $\triangle ABC$. AB is produced to S . D and F are the x - and y -intercepts of AC respectively. F is the midpoint of AC and the angle of inclination of AC is α . The equation of AB is $y = kx + 3$ and the equation of AC is $y = 2x - 2$.



- 3.1 Show that $k = \frac{1}{3}$. (1)
 - 3.2 Calculate the coordinates of B , the x -intercept of line AS . (2)
 - 3.3 Calculate the coordinates of C . (4)
 - 3.4 Determine the equation of the line parallel to BC and passing through $S(-15; -2)$. Write your answer in the form $y = mx + c$. (5)
 - 3.5 Calculate the size of \hat{BAC} . (5)
 - 3.6 If it is further given that the length of AC is $6\sqrt{5}$ units, calculate the value of $\frac{\text{Area of } \triangle ABD}{\text{Area of } \triangle ASC}$. (5)
- [22]**

QUESTION 4

In the diagram, the circle centred at $C(2; p)$ is drawn. $A(5; 1)$ and $B(-3; -3)$ are points on the circle. E is the midpoint of AB .

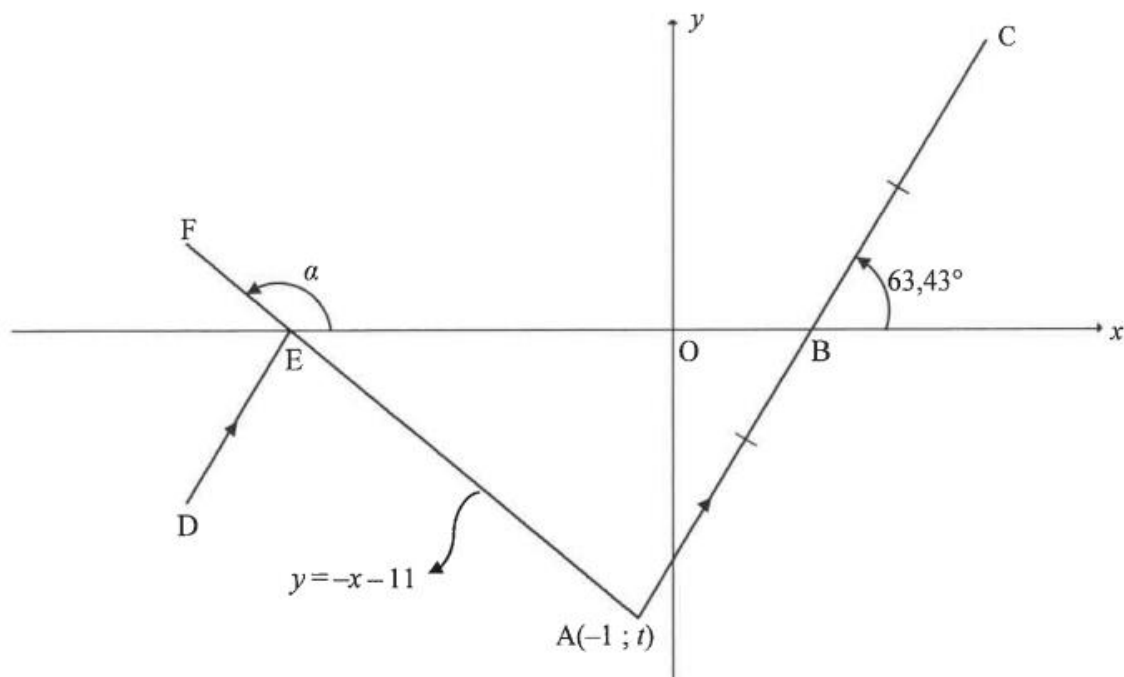


- 4.1 Calculate the coordinates of E , the midpoint of AB . (2)
 - 4.2 Calculate the length of AB . Leave your answer in surd form. (1)
 - 4.3 Determine the equation of the perpendicular bisector of AB in the form $y = mx + c$. (4)
 - 4.4 Show that $p = -3$. (1)
 - 4.5 Show, by calculation, that the equation of the circle is $x^2 + y^2 - 4x + 6y - 12 = 0$ (4)
 - 4.6 Calculate the values of t for which the straight line $y = tx + 8$ will not intersect the circle. (6)
- [18]**

(May/June 2023)

QUESTION 3

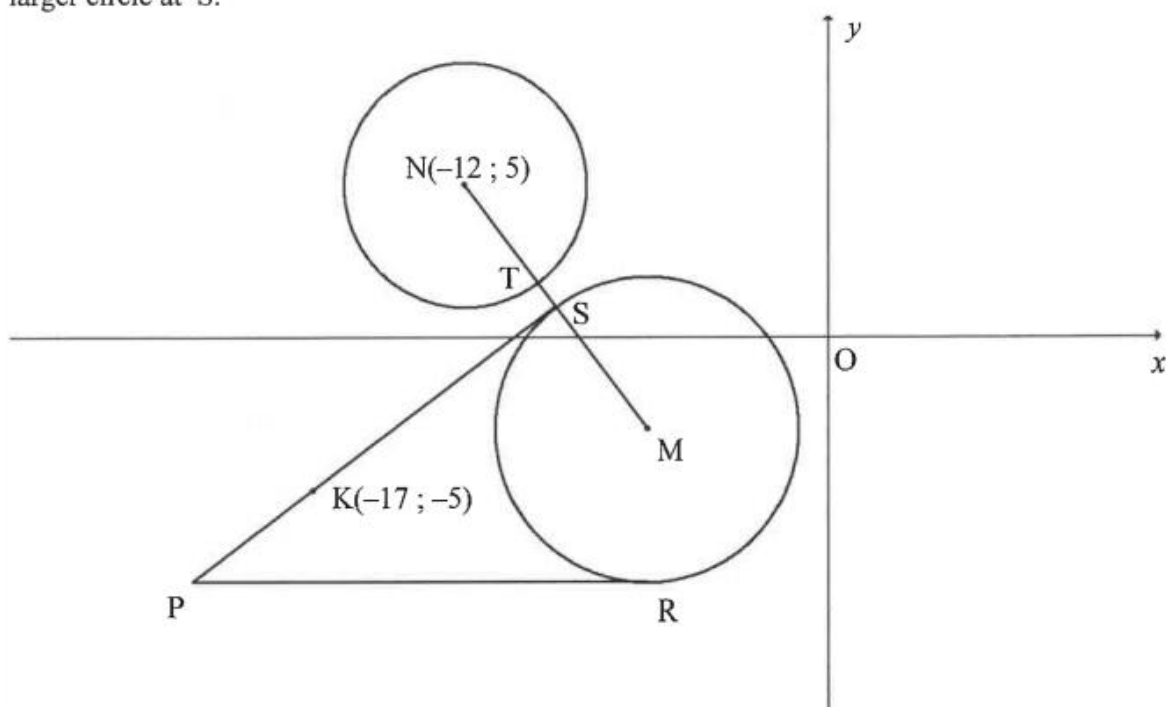
In the diagram, the equation of line AF is $y = -x - 11$. B, a point on the x -axis, is the midpoint of the straight line joining $A(-1; t)$ and C. The angles of inclination of AF and AC are α and $63,43^\circ$ respectively. AF cuts the x -axis in E. D is a point such that $DE \parallel AC$.



- 3.1 Calculate the:
- 3.1.1 Value of t (2)
- 3.1.2 Size of α (2)
- 3.1.3 Gradient of AC, to the nearest whole number (2)
- 3.2 Determine the equation of AC in the form $y = mx + k$. (2)
- 3.3 Calculate the:
- 3.3.1 Coordinates of C (3)
- 3.3.2 Size of \hat{FED} (3)
- 3.4 G is a point such that EAGC, in that order, is a parallelogram.
- Determine the equation of a circle centred at G and passing through the point B.
- Write your answer in the form $(x - a)^2 + (y - b)^2 = r^2$. (4)
- [18]

QUESTION 4

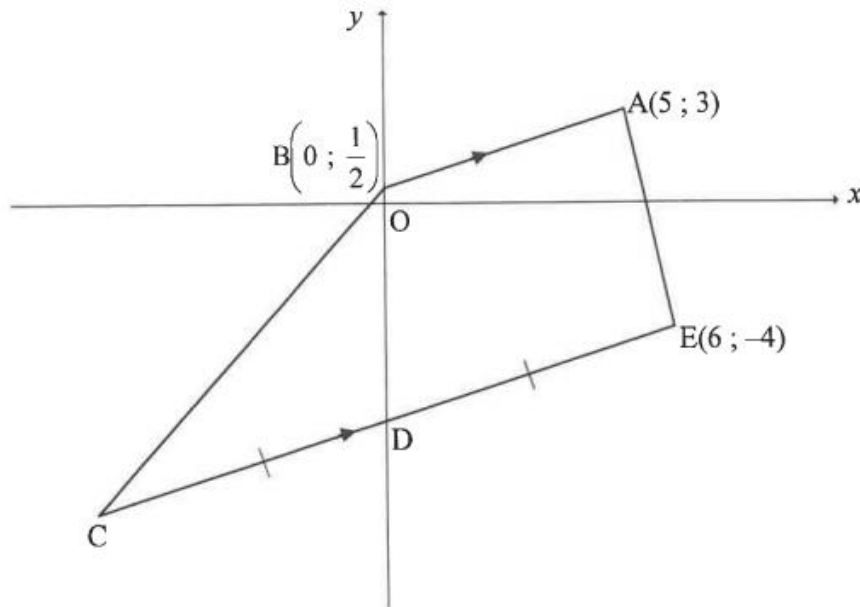
In the diagram, the equation of the circle centred at $N(-12; 5)$ is $x^2 + y^2 + 24x - 10y + 153 = 0$. The equation of the circle centred at M is $(x+6)^2 + (y+3)^2 = 25$. PS and PR are tangents to the circle centred at M at S and R respectively. PR is parallel to the x -axis. $K(-17; -5)$ is a point on PS . The straight line joining N and M cuts the smaller circle at T and the larger circle at S .



- 4.1 Write down the coordinates of M . (2)
- 4.2 Calculate the:
- 4.2.1 Length of the radius of the smaller circle (2)
- 4.2.2 Length of TS (4)
- 4.3 Determine the equation of the tangent:
- 4.3.1 PR (2)
- 4.3.2 PS , in the form $y = mx + c$ (5)
- 4.4 Quadrilateral $PSMR$ is drawn. Calculate the:
- 4.4.1 Perimeter of $PSMR$ (5)
- 4.4.2 Ratio of $\frac{\text{area of } \triangle NPS}{\text{area of quadrilateral } PSMR}$ (2)
- [22]

*(May/June 2022)***QUESTION 3**

In the diagram, $A(5; 3)$, $B\left(0; \frac{1}{2}\right)$, C and $E(6; -4)$ are the vertices of a trapezium having $BA \parallel CE$. D is the y -intercept of CE and $CD = DE$.

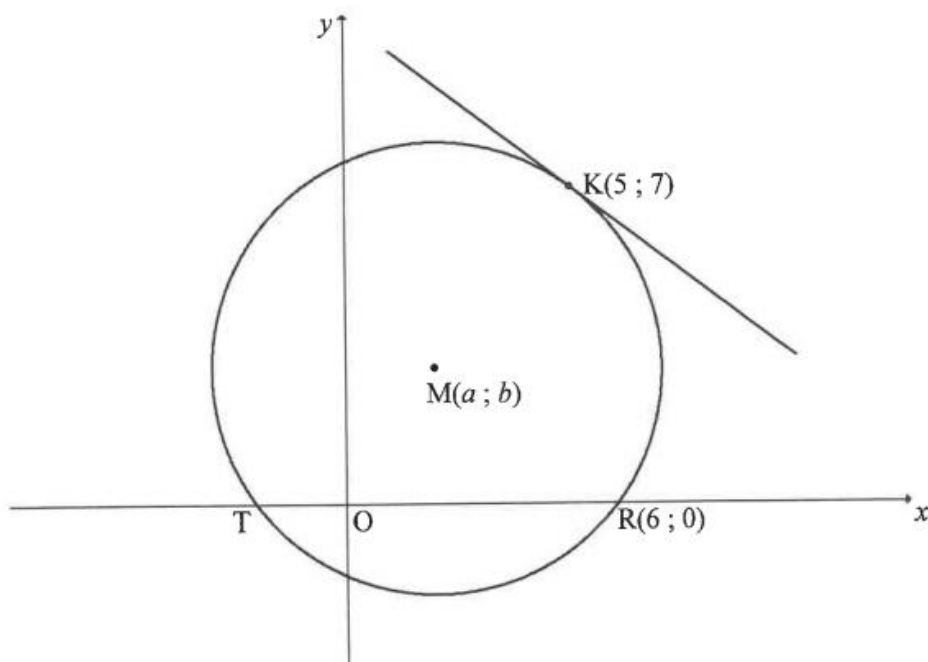


- 3.1 Calculate the gradient of AB . (2)
- 3.2 Determine the equation of CE in the form $y = mx + c$. (3)
- 3.3 Calculate the:
 - 3.3.1 Coordinates of C (3)
 - 3.3.2 Area of quadrilateral $ABCD$ (4)
- 3.4 If point K is the reflection of E in the y -axis:
 - 3.4.1 Write down the coordinates of K (2)
 - 3.4.2 Calculate the:
 - (a) Perimeter of $\triangle KEC$ (4)
 - (b) Size of \hat{KCE} (3)

[21]

QUESTION 4

In the diagram, the circle centred at $M(a; b)$ is drawn. T and $R(6; 0)$ are the x -intercepts of the circle. A tangent is drawn to the circle at $K(5; 7)$.



- 4.1 M is a point on the line $y = x + 1$.
- 4.1.1 Write b in terms of a . (1)
- 4.1.2 Calculate the coordinates of M . (5)
- 4.2 If the coordinates of M are $(2; 3)$, calculate the length of:
- 4.2.1 The radius of the circle (2)
- 4.2.2 TR (2)
- 4.3 Determine the equation of the tangent to the circle at K . Write your answer in the form $y = mx + c$. (5)
- 4.4 A horizontal line is drawn as a tangent to the circle M at the point $N(c; d)$, where $d < 0$.
- 4.4.1 Write down the coordinates of N . (2)
- 4.4.2 Determine the equation of the circle centred at N and passing through T . Write your answer in the form $(x - a)^2 + (y - b)^2 = r^2$. (3)
- [20]**

SECTION 5: INFORMATION SHEET

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$A = P(1 + ni)$$

$$A = P(1 - ni)$$

$$A = P(1 - i)^n$$

$$A = P(1 + i)^n$$

$$T_n = a + (n - 1)d$$

$$S_n = \frac{n}{2} [2a + (n - 1)d]$$

$$T_n = ar^{n-1}$$

$$S_n = \frac{a(r^n - 1)}{r - 1}; \quad r \neq 1$$

$$S_\infty = \frac{a}{1 - r}; \quad -1 < r < 1$$

$$F = \frac{x[(1 + i)^n - 1]}{i}$$

$$P = \frac{x[1 - (1 + i)^{-n}]}{i}$$

$$f'(x) = \lim_{h \rightarrow 0} \frac{f(x + h) - f(x)}{h}$$

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$M\left(\frac{x_1 + x_2}{2}; \frac{y_1 + y_2}{2}\right)$$

$$y = mx + c$$

$$y - y_1 = m(x - x_1)$$

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$m = \tan \theta$$

$$(x - a)^2 + (y - b)^2 = r^2$$

$$\text{In } \triangle ABC: \frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cdot \cos A$$

$$\text{area } \triangle ABC = \frac{1}{2} ab \cdot \sin C$$

$$\sin(\alpha + \beta) = \sin \alpha \cdot \cos \beta + \cos \alpha \cdot \sin \beta$$

$$\sin(\alpha - \beta) = \sin \alpha \cdot \cos \beta - \cos \alpha \cdot \sin \beta$$

$$\cos(\alpha + \beta) = \cos \alpha \cdot \cos \beta - \sin \alpha \cdot \sin \beta$$

$$\cos(\alpha - \beta) = \cos \alpha \cdot \cos \beta + \sin \alpha \cdot \sin \beta$$

$$\cos 2\alpha = \begin{cases} \cos^2 \alpha - \sin^2 \alpha \\ 1 - 2\sin^2 \alpha \\ 2\cos^2 \alpha - 1 \end{cases}$$

$$\sin 2\alpha = 2\sin \alpha \cdot \cos \alpha$$

$$\bar{x} = \frac{\sum x}{n}$$

$$\sigma^2 = \frac{\sum_{i=1}^n (x_i - \bar{x})^2}{n}$$

$$P(A) = \frac{n(A)}{n(S)}$$

$$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$$

$$\hat{y} = a + bx$$

$$b = \frac{\sum (x - \bar{x})(y - \bar{y})}{\sum (x - \bar{x})^2}$$

